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Natural Convection in an Enclosed Cavity with Conjugate Effects Using Fuzzy Logic

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A controller algorithm using a predefined set of fuzzy rules is implemented for fast convergence of a computational fluid dynamics simulation in an enclosed cavity. Relaxation factors are adjusted using the rule set to heuristically guide the iterative scheme towards convergence. The model problem of natural convection in an inclined rectangular cavity is analyzed using this scheme. To test the efficacy of the fuzzy logic guidance method, finite thickness walls with conjugate heat transfer are imposed. Governing equations of continuity, momentum, and energy in laminar flow are solved using a finite difference method. The performance of the controller is compared with that of the simulation with constant relaxation factors. Overall, the number of iterations required for the controller algorithm using fuzzy logic is comparable to the number of iterations obtained using constant relaxation factors. However, six cases of high Grashof number and high tilt angle, which do not converge with any fixed relaxation factor, can be solved using the fuzzy controller. The probability of convergence with the controller is higher than it is when using a fixed relaxation factor.

I. Introduction

LAMINAR natural convection in enclosed cavities with conjugate heat transfer in one or both of the walls has attracted researchers over the past few years. Analysis of these problems is important in a wide range of applications, including building design, design of nuclear reactors, cryogenic systems, furnaces, and solar collectors. Natural convection in enclosures has been studied and recorded in the literature [1–4].

One of the major challenges in the computational fluid analysis of different problems is obtaining convergence in a reasonable number of iterations. Iterative methods form the basis of solving simultaneously the continuity, momentum, and energy equations in fluid flow and heat transfer. The SIMPLER algorithm [5], the basis of the present work, uses simple substitution in order to solve the discretized governing equations of fluid motion, energy, and scalar transport. However, as stated in [6], the success of the iterative method in most computational fluid dynamics (CFD) problems relies on the relaxation of state variables. The optimum relaxation factor depends on the nature of the problem, number of grid points used for discretization, grid spacing, iterative procedure used and other parameters. The optimum relaxation factor cannot be analytically determined. In the relaxation method, the value of the variable to be used for obtaining the solution in the next iteration is the value in the current iteration plus a fraction of the difference between the current value and the predicted value.

Research concerned with using soft computing methods such as fuzzy logic or neural networks to aid CFD simulations are limited in number in the literature. Cort et al. [7] used a simple feedback control method to adjust the relaxation factors in one-dimensional finite element heat transfer simulation. Iida et al. [8] published a study in which wobbling adaptive control was applied to a CFD simulation of the Benard problem. Studies to improve the convergence of genetic algorithms using fuzzy control have been reported in the literature [9–11]. Xunliang et al. [12] controlled the convergence criteria using fuzzy logic based on the residual ratio of momentum or energy equation. A fuzzy logic algorithm for solving turbulent flow conditions is reported in [13].

The relaxation method discussed in the present work enables and improves convergence by slowing down the update rate of the system matrix coefficients. The iterative scheme used in this work is dependent upon the relaxation factor according to the following equation:

$$\phi_p = \phi_p^* + \alpha\left(\frac{\sum a_{nb}\phi_{nb} + b}{a_p} - \phi_p^*\right) \tag{1}$$

where $0 < \alpha < 1$ is the relaxation factor, ϕ_p is the value of the state variable at node P to be used for the next iteration, ϕ_p^* is the value of the state variable at node P in the previous iteration, ϕ_{nb} are the values of the variables at the surrounding nodes and a_p, a_{nb}, and b are the constants from the discretized equation.

The present work deals with the CFD investigation of laminar natural convection in enclosures using fuzzy logic membership functions. These membership functions are adjusted using a predefined set of fuzzy rules. The goal of the work is to test the applicability of the fuzzy control algorithm to a wide range of problems with different geometries. Parametric effect of conductance ratio for different problems analyzed is also investigated. The number of iterations required to obtain a converged solution (considering error ≤ 0.0001) using constant relaxation factors were compared with the iterations needed for the controller algorithm.

II. Problem Analyzed

A. Problem 1: Rectangular Cavity with Conjugate Heat Transfer in One Wall

A schematic of the problem considered is shown in Fig. 1. The problem investigated consists of a rectangular enclosure with isothermal boundary conditions at the extreme sides, i.e., cold (T_c) and hot wall (T_h) conditions at the left and right sides, respectively. The cavity is filled with a constant property fluid and the horizontal sides are insulated. Three walls of the enclosure are assumed to be of negligible wall thickness while the fourth, the right vertical wall, has a thickness t. The thickness ratio (t/L) is kept at 0.2. Because of the temperature gradient along the x direction, a buoyancy-driven

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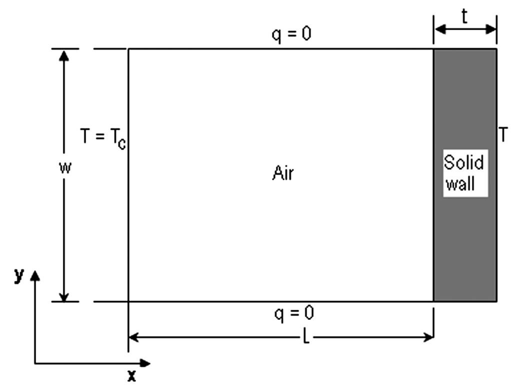


Fig. 1 Rectangular domain with conduction along the right wall.

recirculation pattern appears in the cavity. First, the conjugate heat transfer, i.e., the conduction through the thick walls, is analyzed by considering the conduction at the right side, i.e., hot wall side only. The solid wall at the right side is simulated by substituting a very high value of the dynamic viscosity in the algorithm. The effect of wall conduction on natural convection flow in an enclosure was studied by Balvanz and Kuehn [14]; however, they considered the case of volumetric heat generation within the wall, and the outer face of the thick wall was taken to be insulated. Larson and Viskanta [15] accounted for wall conduction effects in an enclosed fire problem, but only one-dimensional wall conduction was considered. The problem definition and boundary conditions investigated in the present study are different from the preceding studies in that the conjugate effect is taken into account in two-dimensional analysis and for different sides as described in the later sections.

The flow was assumed to be Newtonian, incompressible, laminar, two-dimensional and steady. Viscous dissipation was neglected. All thermophysical properties were assumed constant and independent of the pressure and temperature fluctuations. However, the density was treated using the Boussinesq approximation. The buoyancy force is in the y-direction.

The conservation equations for mass, momentum, and energy are given in Patankar [5] as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2a}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P^*}{\partial x} + \mu\nabla^2 u \tag{2b}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P^*}{\partial y} + g\beta\rho(T - T_{\mathrm{amb}}) + \mu\nabla^2 v \tag{2c}$$

$$\rho c_p\left(u\frac{\partial}{\partial x}(T - T_{\mathrm{amb}}) + v\frac{\partial}{\partial y}(T - T_{\mathrm{amb}})\right) = k\nabla^2(T - T_{\mathrm{amb}}) \tag{2d}$$

where P^* is an effective pressure given by

$$P^* = P + g\rho_c y \tag{3}$$

The velocity components at the boundaries are taken as zero. At the solid–liquid interface, the temperature and the heat flux must be continuous; this condition is mathematically expressed as

$$\left(\frac{\partial\psi}{\partial x}\right)_{\mathrm{fluid}} = \frac{k_w}{k_f}\left(\frac{\partial\psi}{\partial x}\right)_{\mathrm{wall}} \tag{4}$$

where ψ is the nondimensional temperature given by

$$\psi = \frac{T - T_c}{T_h - T_c} \tag{5}$$

and k_w and k_f represent the thermal conductivities of wall and fluid, respectively.

The preceding conservation equations were discretized by a finite volume approach as defined by the following equation:

$$a_p\phi_p = \sum a_{nb}\phi_{nb} + b \tag{6}$$

where b is the source term. The generic variable ϕ is used to represent u, v, and T. Each of the velocity components, temperature and pressure are relaxed by separate relaxation factors.

At every iteration, the assumed values of the solution vector were updated with underrelaxed values according to the following equation:

$$\phi_n^* = \phi_{n-1} + \alpha(\phi_n - \phi_{n-1}) \tag{7}$$

where α is a relaxation factor which varies between 0 and 1. In the present algorithm due to Dragojlovic et. al. [16], the controller adjusts the value of the relaxation factor based on the history of the solution curve for the last 50 iterations. The relaxation factors were applied to the two components of velocity, the temperature and the pressure field. The algorithm comprises of two subalgorithms: one analyzes the nature of the "solution history curve" and the other controls the feature of this curve during iteration in order to produce and preserve those features that bring the fastest convergence. The characteristic quantity that represents the solution at the nth iteration is the square norm of the solution, also known as the magnitude of the solution vector $S^\phi(n)$ defined as

$$S^\phi(n) = \sqrt{\sum_{i=1}^{l}\sum_{j=1}^{m}[\phi_n(i,j)]^2} \tag{8}$$

where i and j are the node numbers in x and y directions, respectively, ϕ_n is the nodal value of the state variable ϕ at the iteration n, l and m are the total number of nodes in the x and y directions, respectively. Tracking the magnitude of the solution vector $S^\phi(n)$ during code execution gives information on how stable and fast the convergence

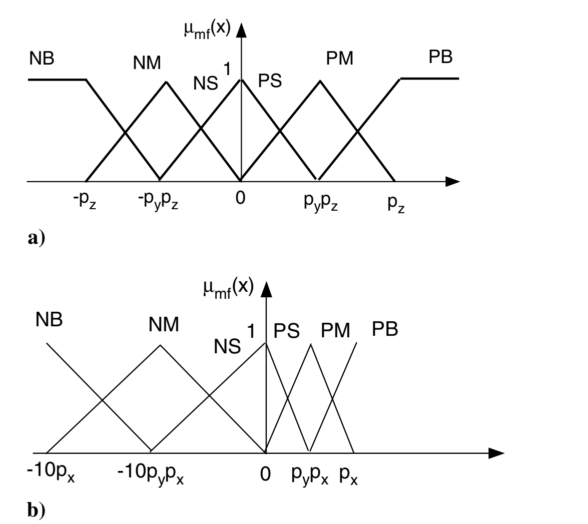


Fig. 2 Membership functions: a) input and b) output; $p_x = 0.000893$, $p_y = 0.6329$, $p_z = 27.785$.

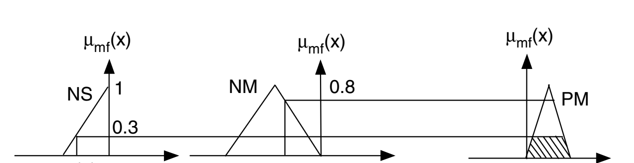


Fig. 3 Example of application of a fuzzy rule.

Table 1 The error is listed to the left of the table and the change in error below the bottom of the table; cells in the table represent the output fuzzy set that results from the size of the error and the change in error

	Change in error					
Error	NB	NM	NS	PS	PM	PB
PB	——	NS	NM	NM	NB	NB
PM	PS	——	NS	NM	NM	NB
PS	PM	PS	——	NS	NM	NM
NS	PM	PM	PS	——	NS	NM
NM	PB	PM	PM	PS	——	NS
NB	PB	PB	PM	PM	PS	——

Table 2 Overall Nusselt numbers at Gr = 10^7 for a variety of grid spacings

Grid size	Overall Nusselt number
20×30	14.68
26×36	14.66
32×42	14.60
38×48	14.55

is and whether divergence is likely to occur. A complete description of the control algorithm is given by Dragojlovic [17].

The algorithm will converge most quickly if the solution history curve is mildly oscillatory. High amplitude oscillations lead to divergence and a monotonic approach to the solution leads to very slow convergence. To determine the degree of oscillation, a discrete Fourier transform of the solution history curve is computed according to

$$H_f = \sum_{k=0}^{N-1} S_k^{\phi} e^{2\pi ikf} \tag{9}$$

Where f, the frequency of the periodic components of the signal, varies between $-1/2$ and $1/2$. The solution vector magnitude S^{ϕ}, was summed over the last N saved values. In the early iterations, N is the total number of prior iterations; after 50 iterations, $N = 50$. After finding the Fourier transform, only the peak values and corresponding frequencies are considered since they identify the main harmonics of the function S^{ϕ} on the given interval.

The only frequencies that have physical meaning are those that vary between 0 and $1/2$. At $f = 0$, the corresponding harmonic does not oscillate at all, while at $f = 1/2$, the harmonic zig-zags on every iteration. The amplitude of the harmonic with frequency f is

$$A_f = \frac{2}{N} \sum_{k=0}^{N-1} S_k^{\phi} e^{2\pi ikf} = \frac{2}{N} H_f = a_f + ib_f \tag{10}$$

Where a_f and b_f are the amplitudes of the cosine and sine function, respectively. The average value of the solution vector on interval N is proportional to the Fourier transform that corresponds to frequency $f = 0$ according to

$$\bar{S}^{\phi} = \frac{1}{N} \sum_{k=0}^{N-1} S_k^{\phi} = \frac{1}{N} H_0 \tag{11}$$

Dividing Eq. (10) by Eq. (11) produces the absolute normalized amplitude of the harmonic with frequency f:

Table 4 Number of iterations for different Gr for problem 1, relaxation factor α

Gr	Iterations				
			$\theta = 0°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	Div	19,499	6314	1344	529
10^5	33,573	9322	3043	656	480
10^7	17,850	4748	1526	670	874
			$\theta = 45°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	Div	19,643	6352	1351	363
10^5	33,813	9289	2976	624	885
10^7	19,817	5361	1806	629	479
			$\theta = 80°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	71,844	19,845	6410	1360	113
10^5	38,631	10,552	3338	677	3626
10^7	25,882	6490	1902	502	547

$$A_f^* = \frac{|A_f|}{\bar{S}^{\phi}} = 2\frac{\sqrt{\mathrm{Re}^2(H_f) + \mathrm{Im}^2(H_f)}}{H_0} \tag{12}$$

From experience, optimal convergence occurs when the amplitudes of high frequency harmonics are very small while the amplitudes of low frequency harmonics are higher. To this end a weighting function designed to highlight the harmonics with high frequencies is defined as

$$W_f(f) = e^{\frac{p_z f}{f_{\max}}} \tag{13}$$

Where $f_{\max} = 0.5$ is the maximum frequency of one cycle per two iterations. The value of p_z was set as 27.785 as recommended by Dragojlovic et al. [16]. The weighting function takes on low values at low frequencies and high values at high frequencies. The goal of the algorithm is to force the weighted amplitudes to be as close to unity as possible, i.e.,

$$\max[W_f(f) A_f^*(f)] \approx 1 \tag{14}$$

Where the left hand side represents the maximum of all the values of $W_f(f)A_f^*(f)$ taken at peaks of the Fourier transform with frequencies f. The control problem was formulated so that the set point is unity, and the error is defined as

$$e(n) = \ln(\max[W_f(f) A_f^*(f)]) \tag{15}$$

The input variables for the control algorithm are the error, $e^{\phi}(n)$ for variable ϕ, and the change in error on iteration n, which is

$$\Delta e^{\phi}(n) = e^{\phi}(n) - e^{\phi}(n-1) \tag{16}$$

The input variables serve as the guiding mechanism for the adjustment of relaxation factors in order to achieve fast convergence. The output variables determined by the control system are the relative changes in relaxation factors defined as

$$\delta^{\phi}(n) = \frac{\Delta\alpha^{\phi}(n)}{\alpha^{\phi}(n)} = \frac{\alpha^{\phi}(n) - \alpha^{\phi}(n-1)}{\alpha^{\phi}(n)} \tag{17}$$

where $\alpha^{\phi}(n)$ is the relaxation factor for the variable ϕ at the nth iteration. Each of the variables $e^{\phi}(n)$, $\Delta e^{\phi}(n)$ and $\delta^{\phi}(n)$ is given a degree of membership in the fuzzy sets used in the rules. The set of

Table 3 Comparison of fuzzy logic algorithm with benchmark problem

Gr	Cratio	Liaqat and Baytas [18] Nu	Kaminski and Prakash [19] Nu	Present Nu
10^3	1.0	0.877	0.87	0.865
10^5	1.0	2.082	2.08	2.09
10^7	1.0	2.843	2.87	2.859

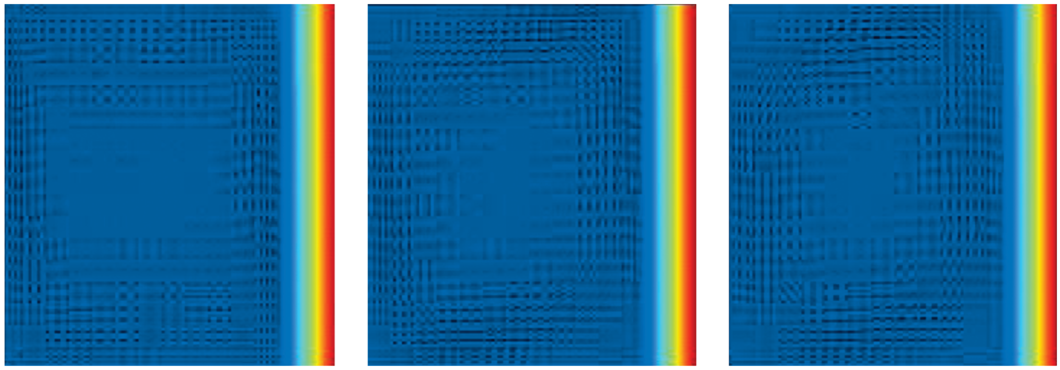


Fig. 4 Temperature distribution and velocity profiles for problem 1 at Gr = 10^7 for different inclination angles.

rules are governed by the fuzzy membership functions which vary between 0 and 1, as shown in Fig. 2. Here **P** and **N** represent "positive" and "negative" and **S** and **B** represent "small" and "big," respectively. Membership function are represented by $\mu_{\mathrm{mf}}(x)$ and p_x, p_y, and p_z represent the parameters on the x-axis. The values of p_x, p_y, and p_z were determined by an optimization process as recommended by Dragojlovic et al. [16].

The degree of membership of the input and output variables in a given fuzzy set are based on their actual values. If the variable $e^{\varphi}(n)$ has a degree of membership of 0.75 in the fuzzy set "positive big," for example, this value is the degree of truth to which $e^{\varphi}(n)$ can be considered positive and big in magnitude. The value of 0 would mean that $e^{\varphi}(n)$ is not positive big at all while the value of 1 means that $e^{\varphi}(n)$ fully belongs to the set positive big.

The rule set is a sequence of if/then-type rules that engage the fuzzy linguistic variables in order to mimic the human, qualitative way of making decisions. An example rule used in this algorithm is as follows: if the error $e^{\varphi}(n)$ is negative small and the change in error is negative medium, then the change in the relaxation factor $\delta^{\varphi}(n)$ is positive medium.

The preceding rule is graphically represented in Fig. 3. In this case, the error is small and getting smaller since the last iteration, therefore the relaxation factor can be moderately increased to speed convergence. The output $\Delta\alpha^{\varphi}(n)/\alpha^{\varphi}(n)$ is computed as the centroid of the triangular curve representing positive medium (PM). The complete set of rules is given is Table 1.

The validity of the fuzzy logic program was first tested with the benchmark problem similar to the one discussed previously but with the conductance ratio (C_{ratio}) of 1.0 as done in [18,19]. The conductance ratio is defined as

$$C_{\mathrm{ratio}} = \frac{k_w}{k_f} \tag{18}$$

where k_w and k_f are the thermal conductivities of the solid wall and fluid, respectively. A grid refinement study for the case of one vertical solid wall with infinite conductance ratio and Gr = 10^7 is shown in Table 2 [20]. The grid used was structured and nonuniform. Control volumes were spaced close to the walls to resolve momentum and thermal boundary layers. It was concluded that a 40×50 grid would accurately represent the overall heat transfer. A later study [16] employed a grid of 80×90 for this same problem and found that doubling the grid size changed the overall Nusselt number by less than 1%. The streamlines and isotherms were also unchanged on the finer grid.

Table 3 shows the comparison of the results with the benchmark results for different values of Grashof number (Gr). For all Grs studied, the present results are very similar to those of two previous investigators.

The rectangular enclosure was considered for different tilt angles, i.e., the inclination of the acceleration due to gravity with respect to the vertical axis. Three different values of tilt angle $\theta = 0, 45$, and 80° are analyzed. Table 4 shows the number of iterations for $\theta = 0$, 45, and 80°, respectively. Numbers of iterations required for convergence using various constant relaxation factors are compared with those using the fuzzy controller. Relaxation factors are varied from 0.1 (smaller relaxation: more time for convergence) to 0.9 (higher relaxation: quick convergence) for three different values of Gr (10^3, 10^5 and 10^7). The grid size used for this example was 40×34. C_{ratio} was kept at 0.01. Out of the 40 vertical grid lines used for discretizing the x-axis, a disproportionate share of 10 grid lines were used for simulating the solid wall conditions. The grid was packed close to the solid walls and the solid–fluid interface so that the boundary layer could be well resolved.

As seen from Table 4, the controller produces convergence in all cases investigated, sometimes in fewer iterations than any of the fixed

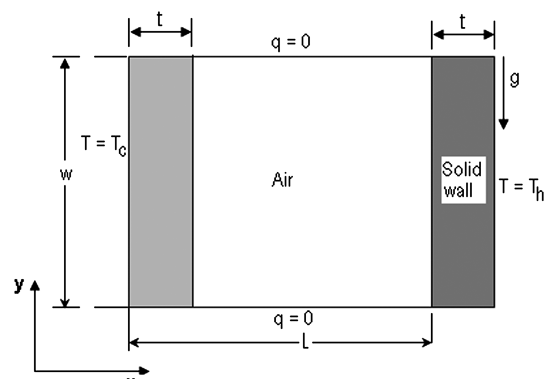
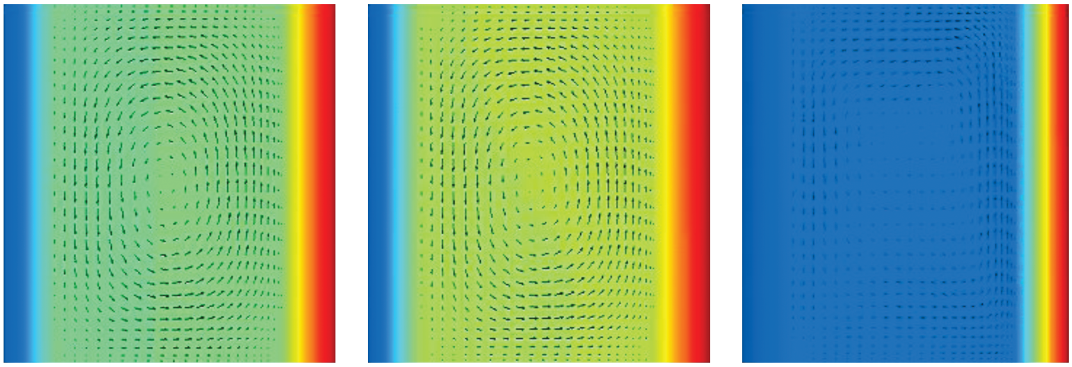


Fig. 5 Rectangular domain with conduction in two side walls.

Table 5 Number of iterations for different Gr for problem 2, relaxation factor α

Gr	Iterations				
	$\theta = 0°$				
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	20,334	5474	1697	344	1145
10^5	22,791	6199	1939	370	394
10^7	29,041	7991	2310	3893	904
	$\theta = 45°$				
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	21,887	5921	1854	388	627
10^5	16,795	4695	1536	311	258
10^7	Div	Div	Div	Div	276
	$\theta = 80°$				
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	22,525	6100	1915	403	513
10^5	20,028	5571	1853	431	511
10^7	Div	Div	Div	Div	386



a) Gr = 10^3 b) Gr = 10^5 c) Gr = 10^7

Fig. 6 Temperature distribution and velocity profiles for problem 2 at zero inclination angle.

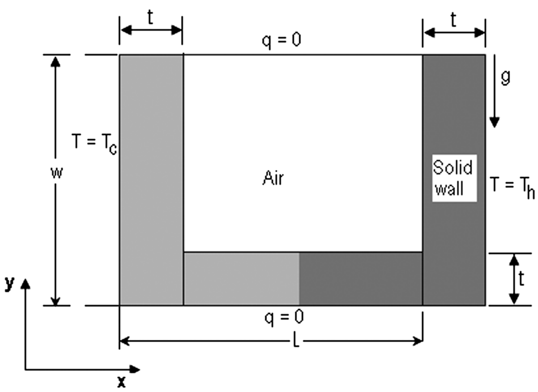


Fig. 7 Rectangular domain with conduction in three walls.

relaxation factors tested and sometimes in more iterations For the case of fixed relaxation factor of 0.1 and Gr of 10^3 with no tilt, the simulation diverges. Divergence probably occurs because, for such low values of relaxation factor, roundoff errors accumulate faster than solution corrections converge. The divergence is defined as the condition when the algorithm fails to find a converged solution in 100,000 iterations. However, the controller was able to achieve convergence in a small number of iterations. The controller operates by selecting small values of relaxation factor early in the iteration to avoid oscillatory divergence and then using larger values later in the iteration to avoid roundoff error induced divergence. The constant relaxation factors fail due to one or the other effect.

Figure 4 shows the temperature and the velocity profiles for the rectangular domain with conjugate heat transfer in the right wall. The figures are the results obtained by the controller algorithm at Grs of 10^7 for different inclination angles. For all tilt angles, the flow pattern consists of one large cell. The boundary layer on the wall opposite the solid wall is thinner in the case of a vertical enclosure, due to the greater driving force. The fluid is largely isothermal, with the bulk of the temperature drop across the solid wall.

B. Problem 2: Rectangular Cavity with Conjugate Heat Transfer in Two Side Walls

This problem is similar to the one analyzed previously with the difference that it involves conjugate heat transfer on two walls, 1 at.the left and the other at the right side, as shown in Fig. 5. The boundary conditions are maintained the same.

Table 5 shows the number of iterations for $\theta = 0$, 45, and 80°, respectively, for the problem under consideration. Interestingly, when the enclosure is tilted at 45 or 80° and $Gr = 10^7$, no constant value of relaxation factor was found that produced convergence.

Figure 6 shows the temperature distribution and the velocity profiles obtained from the controller algorithm for the problem 2 for different Grs at inclination angle of 0°. Note that the temperature field in the solid walls is very one-dimensional, as was the case for the single solid wall. It is in fact not strictly necessary to solve the solid and fluid regions conjugately in this case.

C. Problem 3: Rectangular Cavity with Conjugate Heat Transfer in Three Walls

This problem comprises the conjugate heat transfer in three walls, two side walls and the bottom wall. Figure 7 shows the problem under consideration. The boundary conditions are maintained the same. Also the thickness of the walls is the same.

Table 6 shows the comparison of the number of iterations for different Gr and at different inclination angles. Though the number of iterations was higher in some cases as compared with the constant relaxation factor of 0.9, the controller algorithm was able to meet the convergence criteria irrespective of the intricacy of the problem. For higher values of Gr the controller was able to reach convergence, whereas, the constant relaxation factors failed to do so when the enclosure was tilted.

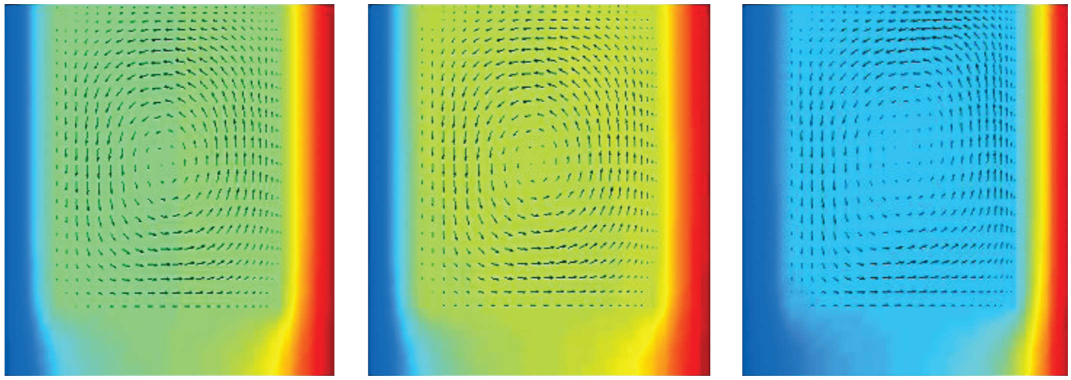
Figure 8 shows the temperature distribution and the velocity profiles for problem 3 for different Grs at inclination angles of 45°. Here the solid wall along the bottom provides an alternate path for heat transfer from hot to cold side. Nevertheless, the wall thermal conductivity is relatively low and a substantial portion of the total heat is transferred by convection. Conjugate effects are more prominent than in the preceding two cases; however, the conduction in the vertical solid walls is still largely one-dimensional in the upper part of the enclosure.

D. Problem 4: Rectangular Cavity with Conjugate Heat Transfer in All Walls

This problem involves the conjugate heat transfer in all four walls. The boundary conditions are the same as described for the preceding cases. Also equal number of grid lines was used to discretize all the

Table 6 Number of iterations for different Gr for problem 3, relaxation factor α

Gr	Iterations				
			$\theta = 0°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	18,625	5017	1555	318	584
10^5	15,485	4198	1305	252	304
10^7	18,236	5230	1579	3582	418
			$\theta = 45°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	18,504	4984	1551	325	153
10^5	14,471	3984	1261	246	425
10^7	Div	Div	Div	Div	259
			$\theta = 80°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	16,882	4561	1425	361	230
10^5	14,057	3780	1163	271	1068
10^7	Div	Div	Div	Div	240



a) Gr = 10^3 b) Gr = 10^5 c) Gr = 10^7

Fig. 8 Temperature distribution and velocity profiles for problem 3 at $\theta = 45°$.

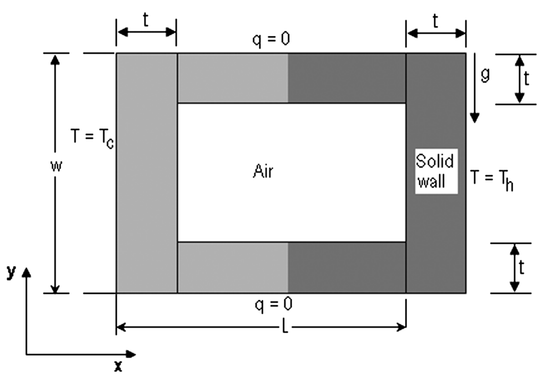


Fig. 9 Rectangular domain with conduction in all the walls.

four walls. A very high value of viscosity number was used to define the wall conditions. Figure 9 shows the problem in consideration. There is a temperature gradient from left wall (at lower temperature) to the right wall (at higher temperature).

Table 7 shows the comparison of the number of iterations for different Gr and at different inclination angles. The performance of controller algorithm exceeded that of the constant relaxation factors for most of the cases. Even in cases where the manual relaxation factors failed to converge, the controller was able to find the result.

Figure 10 shows the temperature distribution and the velocity profiles for problem 4 at inclination angle of 80°. The conduction through the upper and lower solid walls is prominent in this case, although conduction at the midplane of the vertical solid walls is still largely one-dimensional. Flow consists of one large cell without pronounced boundary layers, typical of this high tilt angle.

Table 7 Number of iterations for different Gr for problem 4, relaxation factor α

Gr	Iterations				
			$\theta = 0°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	14,802	3969	1224	251	72
10^5	9997	2706	839	165	285
10^7	9370	2455	985	2979	197
			$\theta = 45°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	14,847	3980	1227	251	68
10^5	11,622	3159	977	182	252
10^7	Div	Div	Div	Div	196
			$\theta = 80°$		
	α 0.1	α 0.3	α 0.6	α 0.9	α controller
10^3	14,860	3984	1229	252	160
10^5	12,858	3687	1188	234	1755
10^7	Div	Div	Div	Div	272

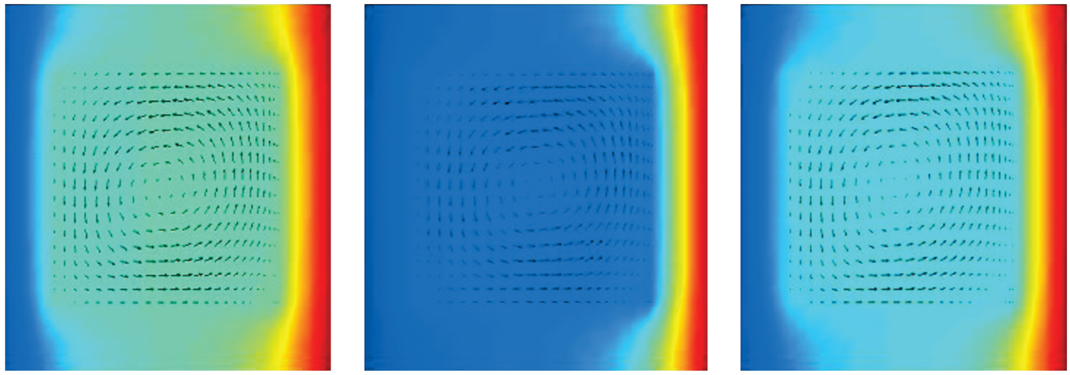
III. Parametric Effect of Conductance Ratio

The controller algorithm was tested for different conductance ratios varying from 0.01 to 10 (0.01, 0.1, 1 and 10). Table 8 shows the average number of iterations for problems 1 to 4 described previously for different relaxation factors and controller algorithm. The average was taken of the iterations that converged for a particular problem. The table also compares the probability of convergence (in percentage) obtained by the constant relaxation factors and the controller algorithm.

It can be inferred from the table that a constant relaxation factor of 0.9 gives a lower average number of iterations than either the other constant values of relaxation factors or the controller in the problems considered. However, the probability of convergence when using 0.9 as the constant relaxation factor is far lower than the other values. In addition to this the controller showed a 100% convergence probability (except for the case when conjugate heat transfer was considered in three walls), i.e., the controller was able to find the converged solution for different sets of problems where the constant relaxation factors failed to do so. These conclusions are, of course, problem dependent. It is reasonable to conclude that, in general, the controller extends the range of convergence to a larger parametric space and

Table 8 Average number of iterations and convergence rate at different relaxation factors and controller algorithm for $\theta = 0°$

Relaxation factor	Average number of iterations	Probability of convergence, %
	Problem 1	
0.1	16,088	86.1
0.3	5439	91.6
0.6	1829	91.6
0.9	489	77.8
Controller	490	100
	Problem 2	
0.1	11,212	94.4
0.3	3210	88.8
0.6	1117	83.3
0.9	440	69.4
Controller	459	100
	Problem 3	
0.1	11,656	94.4
0.3	3394	94.4
0.6	1228	83.3
0.9	521	72.2
Controller	549	97.2
	Problem 4	
0.1	10,864	83.3
0.3	3061	83.3
0.6	1140	83.3
0.9	456	72.2
Controller	643	100



a) Gr = 10^3 b) Gr = 10^5 c) Gr = 10^7

Fig. 10 Temperature distribution and velocity profiles for problem 4 at $\theta = 80°$.

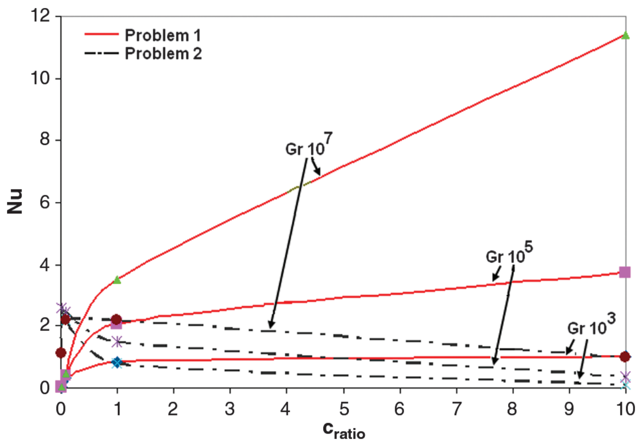


Fig. 11 Nusselt number vs varying conductance ratio for problems 1 (solid line) and 2 (dashed line).

gives a higher probability of quick convergence than using manually set relaxation factors.

A plot of Nusselt number obtained at different conductance ratio for Grs of 10^3, 10^5 and 10^7 is shown in Fig. 11 (problems 1 and 2) and Fig. 12 (problems 3 and 4). The dashed lines in the Figs. 11 and 12 represent problems 2 and 4, respectively.

Except for the case when conjugate heat transfer is considered in one wall the Nusselt number decreases on increasing the conductance ratio from 0.01 to 10. This is because in cases where conjugate heat transfer occurs in more than one wall the driving force for convection is reduced. With higher conductance ratios the conduction through the walls dominates over the internal natural convection which decreases the Nusselt number.

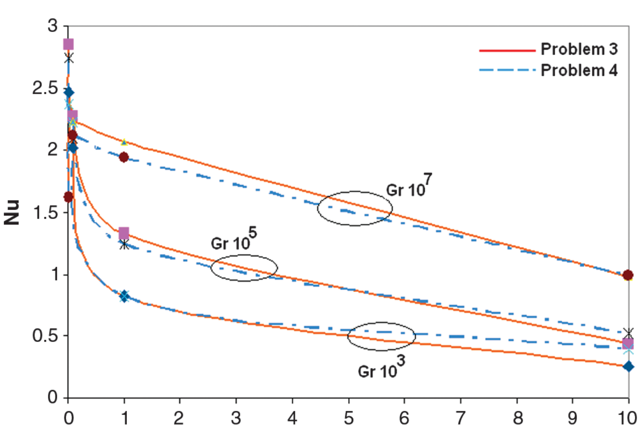


Fig. 12 Nusselt number vs varying conductance ratio for problems 3 (solid line) and 4 (dashed line).

IV. Conclusions

The present work explores the applicability of fuzzy logic convergence control to problems in flow and heat transfer simulation. Different types of conjugate heat transfer problems dealing with natural convection in rectangular enclosures have been examined. Convergence properties of CFD solvers are highly dependent on the choice of the relaxation factors for the variables involved. For a particular problem, a small difference in the value of the relaxation factor can result in a large difference in the number of iterations needed to obtain convergence. Based on the value of the iterative errors the fuzzy set of rules changes the values of the relaxation factors to obtain better convergence. The control algorithm was linked with the sequential solver named SIMPLER, which solves the partial different equations of fluid flow and heat transfer. Four different types of conjugate heat transfer problems dealing with natural convection in rectangular enclosures have been examined. The number of iterations obtained with the controller algorithm is compared with the number of iterations obtained by constant relaxation factors. The number of iterations needed by the controller algorithm to meet the convergence criteria was, on average, a little more than for the case of a fixed relaxation factor of 0.9. The great advantage of the controller is that it converges for some cases where no fixed relaxation factor produced convergence. This was particularly the case with Gr $= 10^7$, where many cases could not be solved with a constant relaxation factor. Parametric effect by varying the conductance ratio for different problems was investigated. The controller algorithm was able to converge with high probability at low number of iterations for the four different problems analyzed. Nusselt number shows an increasing trend with increase in the conductance ratio for the problem having conjugate heat transfer in one wall only.

Although this study was focused on flow and heat transfer, the techniques of fuzzy adjustment of relaxation factors could be extended to other equations of mathematical physics that rely on iterative solution techniques, such as simulation of electromagnetics, solid mechanics, atomic structure, etc. Nothing in the character of the fuzzy controller limits use to flow and heat transfer. In addition, a controller of this type could be used in transient simulation for intelligent choice of time step.

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